Reg. No. :

Question Paper Code : X20776

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
First Semester
Civil Engineering
MA 6151 – MATHEMATICS – I
(Common to all Branches except Marine Engineering)
(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A.

2. If λ is the eigenvalue of the matrix A, then prove that λ^2 is the eigenvalue of A^2.

3. Distinguish between a sequence and series.

4. State the Integral test.

5. Find the radius of curvature of the curve $xy = c^2$ at (c, c)

6. Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$, m being the parameter.

7. If $x^2 + y^2 = 1$, then find $\frac{dy}{dx}$.

8. If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(r, \theta)}{\partial(x, y)}$.

9. Evaluate : $\int_0^{\pi} \int_0^{\sin\theta} r \, dr \, d\theta$.

10. Evaluate $\int_1^3 \int_3^4 \int_1^4 xyz \, dxdydz$.

PART – B **(5×16=80 Marks)**

11. a) i) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$. **(8)**

ii) Using Cayley-Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. **(8)**

(OR)

b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to canonical form. **(16)**

12. a) i) Examine the convergence and the divergence of the following series $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + ... + \frac{2^n - 2}{2^n + 1}\left(x^{n-1}\right) + ...(x > 0)$. **(8)**

ii) Discuss the convergence and the divergence of the following series $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + ...$ **(8)**

(OR)

b) i) Test the convergence of the series $\sum_{n=0}^{\infty} ne^{-n^2}$. **(8)**

ii) Test the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + ...(0 < x < 1)$. **(8)**

13. a) i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, considering it as the envelope of its normals. **(8)**

ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by $a^2 + b^2 = c^2$, c being a constant. **(8)**

(OR)

b) i) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$. **(8)**

ii) Find the circle of curvature at (3, 4) on $xy = 12$. **(8)**

14. a) i) Expand $e^x \log(1 + y)$ in powers of x and y upto the third degree terms using Taylor's theorem. **(8)**

ii) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **(8)**

(OR)

b) i) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. **(8)**

ii) If w = f (y – z, z – x, x – y), then show that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0$. **(8)**

15. a) i) By changing the order of integration, evaluate : $\int_0^1\int_y^1 \frac{x}{x^2+y^2}\,dxdy$. **(8)**

ii) Find the volume of $x^2 + y^2 + z^2 = r^2$ using triple integral. **(8)**

(OR)

b) i) Using double integral, find the area of r = a (1+ cos θ). **(8)**

ii) Evaluate : $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$. **(8)**